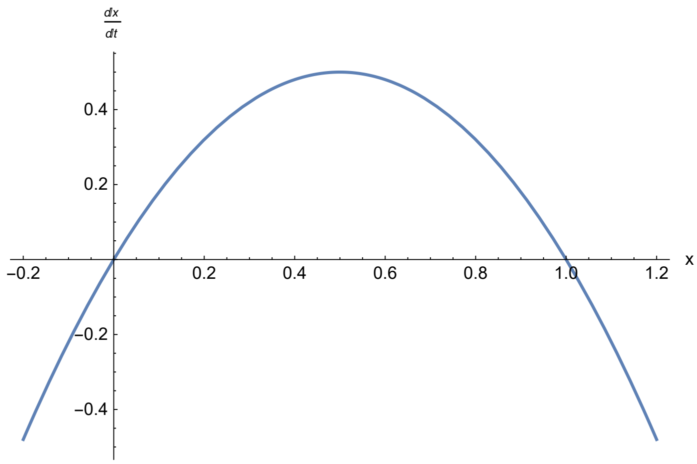
KTO modelo logísitico contínuo

$\frac{dx}{dt} = k\,x(1-x)$



Pontos fixos:

$x_0 = 0 \to \frac{d}{dx}\frac{dx}{dt} > 0$ -> Ponto Fixo repulsivo

$x_1 = 1 \to \frac{d}{dx}\frac{dx}{dt} < 0$ -> Ponto Fixo atrativo

Exercício 0: Calcule x(t) próximo aos pontos fixos:

$$\frac{dx}{dt} = H\left(\hat{x} + \delta x\right) \simeq H(\hat{x}) + \frac{d}{dx}H\,|_{x=\hat{x}}(x-\hat{x})$$

a) $\hat{x} = 0$

$$\frac{dx}{dt} \simeq \frac{d}{dx}H\,|_{x=0}\; x = k(1-2x)\,|_{x=0}\;\delta x = k\delta x$$

$$\frac{dx}{dt} \simeq kx \;\; \text{->} \;\; \delta x(t) = \delta x(0)e^{kt}$$

b) $\hat{x} = 1$

$$\frac{dx}{dt} \simeq \frac{d}{dx}H\,|_{x=1}\; x = k(1-2x)\,|_{x=1}\;(x-1)$$

$$\frac{dx}{dt} \simeq -k(x-1) = \;\; \text{->} -k\delta x \;\; \delta x(t) = \delta x(0)e^{-kt}$$

Exercício 1: Calcule x(t)

$$\frac{dx}{dt} = k\,x(1-x)$$

$$\frac{dx}{x(1-x)} = k\,dt$$

Integrando entre 0 e t, para $x_0 \neq 0,1$

$$\int_{x_0}^{x} \frac{1}{x(1-x)}\, dx = \text{kt}$$

$$\int_{x_0}^{x} \left(\frac{1}{x} + \frac{1}{(1-x)}\right) dx = \text{kt}$$

$$\text{Log}\left[\frac{x}{x_0}\right] - \text{Log}\left[\frac{1-x}{1-x_0}\right] = \text{kt}$$

$$\frac{x}{x_0}\,\frac{1-x_0}{1-x} = e^{\text{kt}}$$

$$\frac{x}{1-x} = \frac{x_0}{1-x_0} e^{\text{kt}}$$

$$= \frac{\left(\frac{x_0}{1-x_0}\right) e^{\text{kt}}}{\left(\frac{x_0}{1-x_0}\right) e^{\text{kt}} + 1}$$

$$\text{x(t)} = \frac{1}{1+\left(\frac{1-x_0}{x_0}\right) e^{-\text{kt}}}$$

$\text{x}(0) = x_0$

$\text{x}(\infty) = 1$

In[59]:=
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g = 0.05; Plot[{1/(1 + (1-g)/g e^-t), 1}, {t, 0, 10}, PlotRange → {0, 1.1}]
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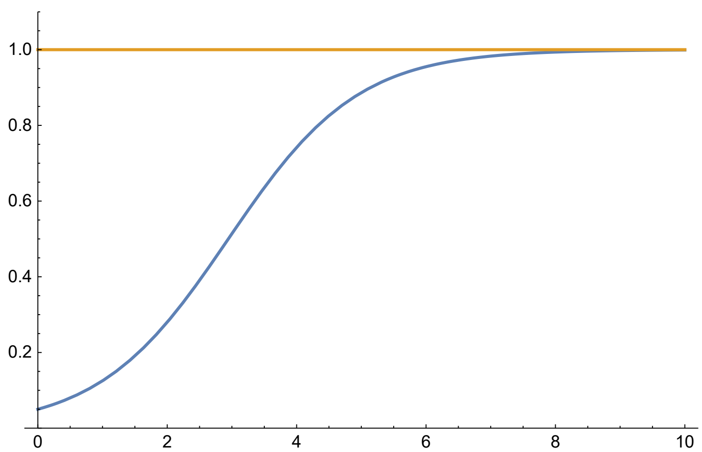
Out[59]=

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Plot[2 x (1 - x), {x, -0.2, 1.2}, AxesLabel → {"x", "dx/dt "}]
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